**Assignment 01**

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1. **Q1.** Algebraically prove that if A ⊂ and C ⊂ then A∩C = φ if A∩∩ = φ, where ∩≠ φ and ⊄, ⊄.

**Answer:**

Given,

1. A ⊂
2. C ⊂
3. A∩∩ = φ
4. ∩≠ φ
5. ⊄ and
6. ⊄

Since A ⊂ and C ⊂ any element in the intersection of A and C must belong to both

​and ​ . If A∩C is non-empty, then there exists an element x such that x∈A and x∈C.

However, from the given condition A∩∩ = φ, it follows that there can be no element that belongs simultaneously to A, and .

Now, let's consider the case where A∩C is not empty. This implies that there exists an element x such that x∈A and x∈C. Since A⊂​ and C⊂​, it follows that x∈​ and x∈.

However, this contradicts the given condition A∩∩= ∅, as we have found an element x that belongs to all three sets A, , and , which is not possible.

Therefore, A∩C must be empty. Thus, algebraically, it is proven that if A∩∩= ∅, where ∩ = ∅ and ⊈ and ⊈​, then A∩C=∅.

1. **Q2.** Assume that P(, ) is a proposition. Prepare a set S using P(, ) such that φ=S.

**Answer:**

A proposition is a statement that can be either true or false, but not both.

To construct the set S such that ∅ = S, we need to find values for and such that P(, ) is always false, resulting in an empty set.

Lets say,

P(, ) = + < 0 Where and are positive number

Then, our set S would be defined as:

S = {(, ) | P(, ) is true}

However, since there are no values of and that satisfies this condition (because the sum of two real positive numbers cannot be less than 0), the set S will be empty.

1. **Q3.** Prove that if A and B are two sets such that, propositions P(x∈A) and Q(x∈B) are tautologies whereas P(x∈B) and Q(x∈A) are contradictions, then A ∩ B = φ.

Given:

Proposition P(x∈A) is a tautology.

Proposition Q(x∈B) is a tautology.

Proposition P(x∈B) is a contradiction.

Proposition Q(x∈A) is a contradiction.

To prove: A∩B = ∅

Let's proceed with the proof:

Since P(x∈A) is a tautology, any element x that belongs to set A satisfies proposition P(x). Similarly, since Q(x∈B) is a tautology, any element x that belongs to set B satisfies proposition Q(x).

However, if P(x∈B) is a contradiction, there cannot exist an element that belongs to both sets A and B. Similarly, if Q(x∈A) is a contradiction, there cannot exist an element that belongs to both sets A and B.

Therefore, there are no elements that simultaneously belong to both sets A and B, implying that the intersection of sets A and B is empty.

Mathematically, A∩B=∅.

Hence, we have proven that ∅=A∩B.